LECTURE NOTES: 4-7 OPTIMIZATION (PART 1)

QUESTION 1: What does *optimization* mean?

QUESTION 2: Where might you encounter the need for optimization or where have you already encounter this?

QUESTION 3: Is there anything wrong with a student who finds $2 \cdot 3$ by explaining:

I find $2 \cdot 3$ *by adding the two numbers then adding 1 to get 6.*

since the student got the right answer?

A CAREFUL LOOK AT THE GOALS OF THIS SECTION:

A MODEL PROBLEM: TWO WAYS Find two positive numbers whose sum is 110 and whose product is a maximum.

PRACTICE PROBLEMS:

1. A rancher has 800 feet of fencing with which to enclose three adjacent rectangular corrals. What dimensions should be used so that the enclosed area will be a maximum?

   (a) Draw and label with numbers two possible fencing arrangements of the type described in the problem and calculate the enclosed area for each.

   (b) Draw and label *with appropriate symbols* the general fencing arrangement.

   (c) Write an expression for the total enclosed area using your choice of symbols. Why are you asked to write an expression for *area* and not something else like perimeter or length or volume, etc?

   (d) Write area as a function of *one* variable. Why is this step important? What is the *domain* of your function?

   (e) Finish the problem by finding the maximum. Show your work in an organized fashion, clearly justifying each step.

   (f) Is your answer reasonable? Explain.

2. An open box of maximum volume is to be made from a square piece of material, 30 inches on a side, by cutting equal squares from the corners and turning up the sides. How should you cut out the corners so that the box has maximum volume?

   (a) Draw and label with numbers two possible choices of cut-out squares and volumes of resulting constructed boxes.

   (b) Draw and label *with appropriate symbols* the general cut-out situation.

   (c) Write an expression for the total enclosed _____ using your choice of symbols.

   (d) Write _____ as a function of *one* variable. What is the *domain* of your function?

   (e) Finish the problem by finding the maximum. Show your work in an organized fashion, clearly justifying each step.

   (f) Is your answer reasonable? Explain.

3. A manufacturer wants to design an open box having a square base and a surface area of 108 square inches. What dimensions will produce maximum volume?

4. A rectangular page is to contain 24 square inches of print. The margins at the top and bottom are to be 1.5 inches, and the margins on the left and right are to be 1 inch. What should the dimensions of the page be so that the least amount of paper is used?

5. Which points on the graph of $y = 4 - x^2$ are closest to the point $(0, 2)$? (Hint: Minimize the distance *squared*.)